Effects of Vertical Wind Shear on Inner-Core Thermodynamics of an Idealized Simulated Tropical Cyclone

JIAN-FENG GU, ZHE-MIN TAN, AND XIN QIU

Key Laboratory of Mesoscale Severe Weather/MOE, and School of Atmospheric Sciences, Nanjing University, Nanjing, China

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ABSTRACT

A suite of idealized simulations of tropical cyclones (TCs) with weak to strong vertical wind shear (VWS) imposed during the mature stage was employed to examine the effects of VWS on the inner-core thermodynamics and intensity change of TCs using a three-dimensional full-physics numerical model as well as a budget analysis of moist entropy. For sheared TCs with shear-induced convective asymmetries, VWS tends to reduce moist entropy within the midlevel eyewall and the boundary layer (BL) but supply moist entropy outside the eyewall above the BL. Such changes in moist entropy reduce the radial gradient of moist entropy across the eyewall, resulting in weakening of the TC. Budget analysis showed that the intense eddy fluxes are mainly responsible for the reduction and/or increase in entropy in the sheared TCs. The entropy reduction within the midlevel eyewall is a result of both the radial eddy flux and the vertical eddy flux. These eddy fluxes are effective at introducing low-entropy air into the midlevel eyewall. Accompanying the flushing of midlevel low-entropy air into the BL, there is an increase in moist entropy outside the eyewall above the BL due to the upward transport of moisture from the BL by shear-induced convection. This represents a new potential pathway to further restrain the radial gradient of moist entropy across the eyewall and hence TC intensity in the sheared environmental flow.

1. Introduction

One of the barriers to improved forecasting of the intensity of tropical cyclones (TCs) is our incomplete understanding of the dynamics of TC–environment interactions, of which the effect of vertical wind shear (VWS) on TC intensity is widely recognized as important one (Wang and Wu 2004). It is well known that strong VWS has a detrimental effect on the intensity of TCs, while weak VWS favors TC genesis (Gray 1968; Tuleya and Kurihara 1981). However, the associated mechanisms are not well understood.

Various theories, consisting of both the dynamical and thermodynamic components, have been proposed to explain changes in TC intensity and structure caused by VWS. The dynamic aspect focuses on the tilt of the inner core by the environmental flow (Jones 1995, 2000; Reasor and Montgomery 2001; Reasor et al. 2004; Schecter et al. 2002) and the subsequent kinematic effects (Wong and Chan 2004; Wu and Braun 2004) on intensity caused by the associated asymmetries. Although the dynamical perspective plays an important role in the evolution of TCs, the thermodynamic cycle is crucial in extracting energy from the ocean to drive the mechanical energy generation required for the development of TCs.

The heat engine is a generally accepted thermodynamic model of TC energetics (Emanuel 1986), in which sea surface enthalpy serves as the energy source, and the work done by the heat engine is converted to kinetic energy to overcome the frictional dissipation. Any process that disrupts the heat engine will inhibit TC development. Thus, the impact of VWS on TC intensity from a thermodynamics perspective concentrates on the constraint of the heat engine by the intrusion of low entropy from the environment. Therefore, one may consider how the low-entropy air in the environment can affect the inner-core thermodynamics of TCs.

Simpson and Riehl (1958) completed the first study of the thermodynamic effects of VWS on TC intensity, and they hypothesized that the midlevel environmental cold
air could ventilate inner-core convection through storm-relative flows. Tang and Emanuel (2010, hereinafter TE10, 2012) suggested two possible pathways through which the VWS could constrain TC intensity. The first was the dilution of eyewall entropy through the radial eddy flux at the midlevel. The second was the flushing of low-entropy air into the boundary layer (BL) inflow by downdraft outside the eyewall. The stronger the magnitude and the lower the height of the midlevel ventilation, the more effective is the disruption of the heat engine (Tang and Emanuel 2012). They suggested that the most destructive mechanism is the midlevel pathway because the heat flux from the ocean surface could offset the reduction of entropy in the BL. Is the radial eddy flux the only way that the inner core at the midlevel can be ventilated? This question arises because their hypothesis is based on the parameterization of shear-induced radial eddies in an axisymmetric model. Given that VWS can induce asymmetries not only for the radial flows, but also for the vertical motions along the azimuth, an axisymmetric model is not sufficient to investigate all aspects of midlevel ventilation. Therefore, a full investigation of midlevel ventilation in a three-dimensional (3D), full-physics model is necessary.

Although the midlevel pathway is emphasized, Riemer et al. (2010, hereinafter RMN10, 2013) pointed out that the second pathway might play the dominant role in the intensity modification by VWS. They demonstrate a close connection between the low-entropy downdraft and the evolution of storm intensity. In their conceptual model, a quasi-stationary convective asymmetry associated with the balanced dynamics and vortex Rossby waves (Montgomery and Kallenbach 1997; Reasor and Montgomery 2001; Reasor et al. 2004) is maintained in the downshear-left quadrant and builds up a low-entropy reservoir in the BL. The heat flux from the ocean cannot completely offset the low-entropy downdraft before the air parcels reach the eyewall, and this leads to a decrease of the entropy in the eyewall. This implies the disruption of the TC heat engine such that less kinetic energy is generated and a decrease in TC intensity can be expected. Despite the strong downdraft at downshear-left, balanced dynamics suggests that the VWS could initiate convection from the downshear-right quadrant outside the eyewall (Jones 1995). While observational studies (Corbosiero and Molinari 2003; Reasor et al. 2009; Reasor and Eastin 2012; Reasor et al. 2013; Zhang et al. 2013; DeHart et al. 2014) have also confirmed the presence of shear-induced convective asymmetries in the inner core, the roles of the downshear-right updrafts outside the eyewall on entropy evolution, and their impact on TC intensity, have yet to be studied and require further investigation.

Although these paradigms have improved our understanding of TC–VWS interactions, their connections and relative importance are not clear. Therefore, systematic investigations are required to evaluate the ventilation hypotheses in a 3D model with full physics to develop an overall picture of the impact of VWS-induced asymmetries on the inner-core thermodynamics of TCs. From the perspective of a TC’s heat engine, the entropy difference across the eyewall plays an important role in determining the storm’s intensity. However, as far as we know, while there have been observational studies (Molinari et al. 2013; Zhang et al. 2013) of inner-core thermodynamics, there have been few detailed numerical investigations of the moist entropy evolution of TCs and its impact on intensity, especially involving VWS.

Zhang et al. (2002, hereinafter ZLY02) performed a budget analysis of equivalent potential temperature $\theta_e$ to examine the inner-core thermodynamics of Hurricane Andrew (1992). Stern and Zhang (2013a, hereinafter SZ13a) investigated the eye warming through a budget analysis of potential temperature. While the analysis of ZLY02 is comprehensive and provides much insight into the inner-core thermodynamics of hurricanes, they are unable to extend our understanding of the thermodynamics of intensity changes driven by VWS simply because their budget analysis on the inner-core thermodynamics was performed at the mature stage during which the hurricane is still intensifying at a rate of 1 hPa h$^{-1}$ and is embedded in a much weaker environmental VWS with 6 m s$^{-1}$ decreasing to 1 m s$^{-1}$. Furthermore, although SZ13a obtained consistent results both in the quiescent and shear environments (Stern and Zhang 2013b), their budget analysis was only conducted for the potential temperature, which is not the most suitable approach for considering energetics. Therefore, in this study, a comprehensive analysis of entropy evolution for a simulated TC in environments with and without shear is presented with the aim of developing a better understanding of the inner-core thermodynamics and paradigms associated with intensity modulation by VWS.

The remainder of the paper is organized as follows. Section 2 describes the model setup and experimental design and gives an overview of the simulated storms. A detailed budget analysis of moist entropy for the investigation of the inner-core thermodynamics is given in section 3. The budget results for the sheared flows are presented together with those of the no-shear case to provide a clearer comparison. In section 4, the processes responsible for the moist entropy change are examined and then the possible impact of moist entropy change on TC intensity is briefly discussed. The main findings are summarized in section 5.
2. Model configuration and overview of simulated storms

a. Numerical model and experimental design

A 3D, nonhydrostatic, full-physics numerical model, the Weather Research and Forecasting (WRF) Model version 3.3.1 (Skamarock et al. 2008), was used to conduct a set of idealized experiments on an f plane valid at 15°N. The model had 35 vertical levels and two domains of 721 × 361 and 181 × 181 grid points, with resolutions of 15 and 5 km, respectively. The inner domain was centered within the outer one at the initial time and moved with the vortex. The Yonsei University (YSU; Hong et al. 2006) and the WRF 5-class single moment (WSM5; Hong et al. 2004) schemes were chosen for the parameterization of planetary boundary layer and microphysics processes, respectively. The Betts–Miller–Janjic (Betts and Miller 1986) cumulus parameterization was used in the outer domain. The sea surface temperature was set to 28.5°C. Although recent studies (Fovell et al. 2010; Bu et al. 2014) showed that radiative forcing can impact storm structure, they focused on the outer-core winds and the subsequent impact on storm track rather than the inner-core intensity. Therefore, the radiative processes were neglected for simplicity in the present study.

The vortex profile chosen for this study follows that of Qiu et al. (2010) and Qiu and Tan (2013). The results may be sensitive to the initial vortex specification; however, this is beyond the scope of this study. The initial vortex was centered in the inner domain and had an axisymmetric structure with a maximum surface wind speed of 20 m s⁻¹ at 135 km. The method proposed by Nolan (2011) was used to initialize the vortex and the baroclinic environment, with the Jordan sounding (Jordan 1958) as the background sounding.

To examine the impact of VWS of different magnitudes on the evolution of storm intensity, three experiments (SH05, SH10, and SH15) were conducted by superimposing a VWS of 5, 10, and 15 m s⁻¹ on the no-shear simulation (SH00) at 120 h when the storm was in its mature stage. The VWS was placed linearly between heights of 2 and 12 km, with zero flow in the lower levels and an easterly flow in the upper levels.

b. Overview of simulated storms

Figure 1a shows the temporal evolution of azimuthally averaged maximum wind speed of the simulated TCs. There is no noticeable effect of vertical wind shear in SH05 compared with SH00, while the storm weakens significantly from the quasi-steady state of the first 12 h after the imposition of moderate to strong VWS (SH10 and SH15), which is consistent with previous studies.

Figure 1b shows the temporal evolution of the azimuthally averaged moist entropy difference between the inner region (0–50 km) and outer region (80–120 km) of the inner core, which is averaged between heights of 2 and 10 km. The change in the azimuthally averaged entropy difference with time coincides well with that of TC intensity, not only in SH00, but also in all of the shear experiments, suggesting a close connection between TC
intensity and the radial gradient of moist entropy across the eyewall, which is in agreement with theoretical studies (Emanuel 1997).

Figure 2 shows the evolution of azimuthally averaged moist entropy from 3 to 8 h after the VWS was imposed as well as the time-averaged vertical velocity during this period. Three distinct characteristics of the strong-VWS cases (SH10 and SH15) distinguish them from the no- and weak-VWS cases (SH00 and SH05). First, the moist entropy in the strong updraft increases at low to mid-levels when the storm remains steady (SH00 and SH05; Figs. 2a,b). However, the moist entropy decreases in the eyewall in the moderate and strong VWS cases (SH10 and SH15; Figs. 2c,d). Second, there is a strong reduction of moist entropy in the BL for the moderate and strong VWS cases (Figs. 2c,d), while it shows little change in the SH00 and SH05 runs (Figs. 2a,b). These two features of the moist entropy evolution in SH10 and SH15 confirm the previous thermodynamic paradigms regarding modification of intensity by VWS (RMN10; TE10). Moreover, another distinct feature, which has not been emphasized previously, is an increase of moist entropy outside the eyewall above the BL in the moderate and strong VWS cases (Figs. 2c,d). Although some increase in moist entropy outside the eyewall above the BL could also be seen in SH00 and SH05 (Figs. 2a,b), their values, and the region of this increase, are smaller. The longer the time scale is, the stronger these features are, especially for the entropy increase outside the eyewall above the BL (not shown). This moist entropy increase outside the eyewall above the BL might also be partly responsible for the decrease in intensity of the TC. A brief explanation of this possibility can be found in section 4c. Therefore, from the axisymmetric point of view (Fig. 2), the moist entropy of a TC decreases both in the midlevel eyewall and the BL but increases outside the eyewall above the BL when the TC is affected by moderate and strong VWS.
3. Moist entropy budget analysis

a. Budget equations

To obtain a clear picture on how the moist entropy of a TC evolves as described above, a detailed budget analysis of moist entropy is required. Furthermore, a budget analysis that includes the ice processes should provide a more accurate picture as our experiments use a microphysics parameterization that includes ice processes.

According to Emanuel [1994; see his (4.5.9)], the moist entropy, including the liquid–ice processes (see the appendix), is given by

\[ S = (c_p + c_l q_i) \ln T - R \ln p_d + \frac{L_v q_v}{T} - \frac{L_f q_l}{T} - R_v (q_v + q_i) \ln(\mathcal{H}_i) + R_q q_i \ln(\mathcal{H}_i), \tag{1} \]

where \( T \) is the absolute temperature; \( p_d \) is the partial pressure of dry air; \( R \) and \( R_v \) are the gas constant for dry air and water vapor, respectively; \( c_p \) is the specific heat at constant pressure for dry air; \( c_l \) is the specific heat of liquid water; \( q_v \), \( q_v \), and \( q_i \) are the mixing ratios of total water, water vapor, and ice, respectively; \( L_v \) and \( L_f \) are the latent heat of vaporization and freezing, respectively, and they are functions of temperature; \( \mathcal{H}_i \) is the relative humidity with respect to water; and \( \mathcal{H}_i \) is the relative humidity with respect to ice.

Following Bryan (2008, hereinafter B08), an approximated formula for the moist entropy including liquid-ice processes can be given as follows:

\[ S = c_p \ln T - R \ln p_d + \frac{L_v q_v}{T} - \frac{L_f q_l}{T} - R_v (q_v + q_i) \ln(\mathcal{H}_i) + R_q q_i \ln(\mathcal{H}_i), \tag{2} \]

where \( L_v \) and \( L_f \) are introduced as constants and are set to \( 2.555 \times 10^6 \) J kg\(^{-1}\) and \( 2.832 \times 10^5 \) J kg\(^{-1}\), respectively. This simplification of the latent heat of vaporization \( L_v \) and freezing \( L_f \) aims to compensate for the ignorance of \( c_l q_i \), for the simplicity of calculation and is acceptable for a wide range of atmospheric conditions. The detailed discussion of the approximation of Eq. (1) is given in the appendix. If the liquid–ice processes are excluded, Eq. (2) is reduced to

\[ S = c_p \ln T - R \ln p_d + \frac{L_v q_v}{T} - R_v q_v \ln(\mathcal{H}_i), \tag{3} \]

which is the formulation of moist entropy in B08.

The governing equations for the moist entropy \( S \) [see the derivation of Eq. (A14) in the appendix], the potential temperature \( \theta \), mixing ratio of water vapor \( q_v \), and mixing ratio of ice \( q_i \), in cylindrical coordinates, are

\[ \frac{dS}{dt} = \mu_1 \frac{d\theta}{dt} + \mu_2 \frac{dq_v}{dt} + \mu_3 \frac{dq_i}{dt} - R_v \frac{d\ln p_d}{dt}, \tag{4} \]

\[ \frac{d\theta}{dt} = \dot{\theta}_{\text{MIC}} + \dot{\theta}_{\text{PBL}} + \dot{\theta}_{\text{DIF}}, \tag{5} \]

\[ \frac{dq_v}{dt} = \dot{q}_{v\text{MIC}} + \dot{q}_{v\text{PBL}} + \dot{q}_{v\text{DIF}}, \tag{6} \]

\[ \frac{dq_i}{dt} = \dot{q}_{i\text{MIC}} + \dot{q}_{i\text{PBL}} + \dot{q}_{i\text{DIF}}, \tag{7} \]

where

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} + v \frac{\partial}{\partial \theta} + w \frac{\partial}{\partial z} \]

and \( \mu_1 = c_p \theta, \mu_2 = (L_v/T) - R_v [\ln(\mathcal{H}_i) + 1], \) and \( \mu_3 = -[(L_f/T) + R_v \ln(\mathcal{H}_i/\mathcal{H}_i)] \). The variables \( u, v, \) and \( w \) are the storm-relative radial, tangential, and vertical velocities, respectively. The subscripts MIC, PBL, and DIF, together with the dot, represent the tendency contributions by the microphysics, planetary boundary layer, and the diffusions, respectively. Using the characteristic values for scaling analysis, the last term on the right-hand side of Eq. (4) is at least one or two orders of magnitude smaller and can be ignored.

The variables are first decomposed into the azimuthal-mean and perturbations. Then, the equations are averaged along the azimuth and any terms two orders of magnitude (or more) smaller are ignored. The azimuthally averaged budget equations for the moist entropy, potential temperature, and mixing ratios of water vapor and ice, in the storm-relative coordinates, are as follows:

\[ \frac{\partial \tilde{S}}{\partial t} \simeq \frac{\partial \tilde{\theta}}{\partial t} + \frac{\partial \tilde{q}_v}{\partial t} + \frac{\partial \tilde{q}_i}{\partial t}, \tag{8} \]

\[ \frac{\partial \tilde{\theta}}{\partial t} = \tilde{\theta}_{\text{ADV}} + \tilde{\theta}_{\text{MIC}} + \tilde{\theta}_{\text{PBL}} + \tilde{\theta}_{\text{DIF}}, \tag{9} \]

\[ \frac{\partial \tilde{q}_v}{\partial t} = \tilde{q}_{v\text{ADV}} + \tilde{q}_{v\text{MIC}} + \tilde{q}_{v\text{PBL}} + \tilde{q}_{v\text{DIF}}, \tag{10} \]

\[ \frac{\partial \tilde{q}_i}{\partial t} = \tilde{q}_{i\text{ADV}} + \tilde{q}_{i\text{MIC}} + \tilde{q}_{i\text{PBL}} + \tilde{q}_{i\text{DIF}}, \tag{11} \]

where

\[ \tilde{\theta}_{\text{ADV}} = - \left( \frac{\partial \tilde{\theta}}{\partial r} + \frac{\partial \tilde{\theta}}{\partial \theta} + \frac{u' \partial \tilde{\theta}}{\partial r} + \frac{w' \partial \tilde{\theta}}{\partial z} \right), \]

\[ \tilde{q}_{v\text{ADV}} = - \left( \frac{\partial \tilde{q}_v}{\partial r} + \frac{\partial \tilde{q}_v}{\partial \theta} + \frac{u' \partial \tilde{q}_v}{\partial r} + \frac{w' \partial \tilde{q}_v}{\partial z} \right), \]
and $\mathbf{\overline{u}} = c_p \mathbf{\overline{\theta}},$ $\mathbf{\overline{v}} = (L_v \mathbf{\overline{T}}) - R_v [\ln (\mathbf{\overline{H}}_t) + 1],\, \mathbf{\overline{w}} = -[(L_{v0} \mathbf{\overline{T}}) + R_v \ln (\mathbf{\overline{H}}_t/\mathbf{\overline{H}}_i)]$. The subscript “ADV” represents the tendency of moist entropy by advection. The overbar represents the azimuthal mean and the prime indicates perturbation. The tangential eddy advection is very small and so is not shown.

From Eqs. (9)–(11), the equation for the tendency of moist entropy [Eq. (8)] could be rewritten as

$$\frac{\partial \mathbf{\overline{S}}}{\partial t} = \mathbf{\overline{S}}_{\text{ADV, M}} + \mathbf{\overline{S}}_{\text{ADV, E}} + \mathbf{\overline{S}}_{\text{PHY}},$$

Fig. 3. Radius–height plot of total moist entropy tendency (J kg\(^{-1}\) K\(^{-1}\) s\(^{-1}\)): (a) TOTAL of SH00 and (b) TOTAL of SH10. The mean advection (RAD_M, VER_M), eddy advections (RAD_E, VER_E), and physics (PHY) tendencies of moist entropy: (c) RAD_M, SH00; (d) RAD_M, SH10; (e) VER_M, SH00; (f) VER_M, SH10; (g) RAD_E, SH00; (h) RAD_E, SH10; (i) VER_E, SH00; (j) VER_E, SH10; (k) PHY, SH00; and (l) PHY, SH10.
\[
\mathcal{S}_{\text{ADV,M}} = - \left( \mu_1 \frac{\partial \bar{\theta}}{\partial r} + \mu_2 \frac{\partial \bar{q}_v}{\partial r} + \mu_3 \frac{\partial \bar{q}_i}{\partial r} \right) \left( \mu_1 \frac{\partial \bar{\theta}}{\partial z} + \mu_2 \frac{\partial \bar{q}_v}{\partial z} + \mu_3 \frac{\partial \bar{q}_i}{\partial z} \right), \tag{13} \]

\[
\mathcal{S}_{\text{ADV,E}} = - \left( \mu_1 \frac{\partial \bar{\theta}'}{\partial r} + \mu_2 \frac{\partial \bar{q}_v'}{\partial r} + \mu_3 \frac{\partial \bar{q}_i'}{\partial r} \right) \left( \mu_1 \frac{\partial \bar{\theta}'}{\partial z} + \mu_2 \frac{\partial \bar{q}_v'}{\partial z} + \mu_3 \frac{\partial \bar{q}_i'}{\partial z} \right), \tag{14} \]
and the results were consistent with this period. Therefore, the analysis of the 5–6-h period following the imposition of VWS is representative and is appropriate for exploring the mechanisms that are responsible for the rapid weakening of a TC in the first 12 h. For convenience, the case SH10 was chosen for detailed discussion as the characteristics of the budget analysis in the shear flows are generally similar. The result is presented together with SH00 (Fig. 3) for comparison and to provide a clear image of the role of VWS in entropy evolution.

Figures 3a–f show the total tendency of moist entropy [Eq. (12)] and the tendency associated with azimuthal-mean advections [RAD_M and VER_M; Eq. (13)] in SH00 and SH10. In the presence of VWS, the total tendency in SH10 demonstrates a reduction in the eyewall and BL and an increase outside the eyewall above the BL (Fig. 3b), while in SH00, the moist entropy increases in the eyewall at the low to midlevels (Fig. 3a). The azimuthal-mean advections of moist entropy in SH00 and SH10 share almost the same features. In the eyewall, the azimuthal-mean radial outflow transports high entropy air outward (Figs. 3c,d) and the strong updraft brings low-entropy air upward (Figs. 3e,f) because the entropy decreases outward and increases with height above the BL. At the bottom of the eyewall, the radial inflow brings low entropy (Figs. 3c,d) inward and the vertical motion transports high-entropy air upward (Figs. 3e,f). Furthermore, in the BL between a radius of 80 and 120 km, the stronger positive radial-mean advection of moist entropy in SH10 indicates that there is a strong low-entropy reservoir. This is because the enhanced positive radial-mean advection of moist entropy could only be achieved by the increased radial gradient of entropy in the BL because the radial-mean advection is \(-\overline{\nabla S}_r\) (shown in Figs. 3g–l). Because of the highly axisymmetric nature of the TC in SH00, the eddy advections (Figs. 3g,i) are much smaller than the mean advections (Figs. 3c,e), especially for the radial eddy advection. However, in the presence of VWS, the eddy advections show very different characteristics (Figs. 3h,j). They not only concentrate in the eyewall as in SH00 but also have an important effect on the moist entropy evolution outside the eyewall and in the BL. The radial eddy advection of moist entropy plays a significant role in depressing the moist entropy in the midlevel eyewall and in the BL between a radius of 40 and 120 km (Fig. 3h). Figure 3j indicates that most of the negative influence of vertical eddy advection (VER_E) is concentrated in the midlevel eyewall from 3 to 6 km in height, while above

\[ \overline{S}_{PHY} = \overline{S}_{MIC} + \overline{S}_{PBL} + \overline{S}_{DIF}, \]  
\[ \overline{S}_{MIC} = \overline{\mu}_1 \overline{\theta}_{MIC} + \overline{\mu}_2 \overline{q}_{IMIC} + \overline{\mu}_3 \overline{q}_{DIF}, \]  
\[ \overline{S}_{PBL} = \overline{\mu}_1 \overline{\theta}_{PBL} + \overline{\mu}_2 \overline{q}_{IPBL} + \overline{\mu}_3 \overline{q}_{IPBL}, \]  
and
\[ \overline{S}_{DIF} = \overline{\mu}_1 \overline{\theta}_{DIF} + \overline{\mu}_2 \overline{q}_{IDIF} + \overline{\mu}_3 \overline{q}_{IDIF}. \]

The subscripts “ADV_M” and “ADV_E” represent the mean and eddy advection of moist entropy, respectively; “RAD_M” and “RAD_E” denote the radial mean and radial eddy advection of moist entropy, respectively; “VER_M” and “VER_E” represent the vertical mean and vertical eddy advection of moist entropy, respectively; and “PHY” denotes the total tendency of moist entropy by MIC, PBL, and DIF. The tendencies associated with MIC, PBL, and DIF were obtained directly from the output of the simulation over the fine-mesh domain at 5-min intervals. The translation speed was subtracted from the horizontal velocities. The variables were transformed from the model grids to the cylindrical grids with the storm center at the origin. The advection terms were calculated in the cylindrical coordinates and then averaged azimuthally and temporally together with the other physical contributions. The storm center was selected as the vorticity centroid within a 100-km radius of the minimum pressure center at a height of 3 km. The vorticity centroid was calculated using 20 iterations. It has been confirmed that the budget results are not sensitive to the choice of height within this radius. Errors may be introduced through various pathways such as the interpolations, the numerical difference method, the output frequency, the movement of the TC, and some missing terms. However, the calculated tendency was qualitatively the same as the result from model both in the magnitude and patterns (not shown), suggesting its reliability for use in physical interpretations.

b. Moist entropy budget in a sheared TC

In the following, the moist entropy budget analysis was applied to the 1-h time interval from 5 to 6 h after the VWS was imposed. The evolution of vortex tilt (not shown) indicates that 5–6 h after the VWS is imposed is enough for the vortex to respond and is also not so long that TC intensity decreases to a relative stable stage. In addition, during this period the moist entropy evolves consistently with the 5-hourly evolution (Fig. 2). A budget analysis was also performed over other time intervals, and the results were consistent with this period. Therefore, the analysis of the 5–6-h period following the imposition of VWS is representative and is appropriate for exploring the mechanisms that are responsible for the rapid weakening of a TC in the first 12 h. For convenience, the case SH10 was chosen for detailed discussion as the characteristics of the budget analysis in the shear flows are generally similar. The result is presented together with SH00 (Fig. 3) for comparison and to provide a clear image of the role of VWS in entropy evolution.

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which the moist entropy is strongly increased by the VER_E. The role of VER_E in reducing moist entropy also has an impact on the inner edge of the eyewall from the BL to a height of 10 km. Moreover, the VER_E reduces the moist entropy in the BL from a radius of 40 km outward to 120 km, with the top of the negative tendency increasing in height from 0.5 km inside to 1.5 km outside. Another distinguishing feature is the positive tendency of the VER_E outside the eyewall above 1.5 km in height, which does not occur in SH00. This positive tendency is mostly the result of the VER_E of water vapor (not shown), indicating the important role of convection outside the eyewall in transporting moisture upward and so moistening the midtroposphere.

Figures 3k and 3l present the total physics tendency associated with the microphysics, planetary boundary layer, and diffusion in SH00 and SH10, respectively. The microphysics tendency is negative in the melting layer and positive in the eyewall above this layer. The PBL tendency is mostly positive in the region of high wind, implying energy extraction from the ocean. Diffusion always depresses the entropy at the inner edge of the eyewall but has a much smaller effect than that of the other two physical terms. The VWS has no obvious effect on the sources and sinks of moist entropy except in the BL outside a radius of 80 km and at heights from 0.5 to 2 km, where the physics tendency is enhanced.

Figure 4 shows the eddy tendency ($\overline{S}_{ADV,E}$; Figs. 4a,b) and noneddy tendency ($\overline{S}_{ADV,M} + \overline{S}_{PHY}$; Figs. 4c,d). The noneddy tendency is defined as the sum of mean advection and the physical tendencies of moist entropy. It is clear that within the eyewall, in the absence of VWS, the noneddy tendency (Fig. 4c) plays a dominant role, resulting in a net increase in moist entropy at the low to midlevels and a decrease in the upper levels (Fig. 3a). There is little moist entropy change outside the eyewall. On the contrary, when the VWS is imposed, the eddy tendency is considerably enhanced (Fig. 4b) in magnitude, leading to the reduction in moist entropy in the midlevel eyewall and in the BL from a radius of 40 km outward (Fig. 3b). Moreover, there is an obvious increase in moist entropy outside the eyewall at a radius from 80 to 120 km, and a height of 2–5 km, which is also contributed by the eddy tendency (Figs. 4b and 3b).

To provide further evidence of the crucial role of shear-induced convective asymmetries in the inner-core moist
entropy evolution, Figs. 5 and 6 show the vertical profile of radially averaged moist entropy tendencies within the eyewall (40–60 km) and outside the eyewall (80–120 km). ADV_M and ADV_E are defined in Eqs. (13) and (14), respectively. The PHY tendency is the sum of the MIC, PBL, and DIF tendencies as in Eq. (15). Compared with SH00 (Fig. 5a), the moist entropy decreases both in the BL and in the midlevel eyewall with a distinct feature that the eddy advection becomes much more important in SH10 (Fig. 5b). The eddy advection is enhanced by VWS throughout most of the troposphere, especially in the BL below 1.5 km and at the midlevels from 4 to 6 km in height. Negative eddy advection in the BL is mostly determined by RAD_E (Fig. 5d) and is much stronger than that in SH00 (Fig. 5c). At midlevels within the eyewall, the VWS induces much stronger asymmetries, which bring the low-entropy air into the eyewall both through RAD_E and VER_E (Fig. 5d).

Outside the eyewall, the tendencies of moist entropy in SH10 (Fig. 6b) are much larger in the low to midlevels than that in SH00 (near zero; Fig. 6a). Of importance is the much more pronounced eddy advection resulting from the shear-induced asymmetries (Fig. 6d) when compared with that in SH00 (Fig. 6c). Similar to the contribution of RAD_E, the VER_E also brings the low-entropy air from the midlevel into the BL. The VWS-induced convection (Fig. 6d) outside the eyewall is the dominant contributor for transporting air with high moist entropy upward from the BL to increase the entropy of the midtroposphere (VER_E). Therefore, the shear-induced eddy fluxes play an important role in the inner-core moist entropy evolution when a TC is affected by VWS.

4. Change of moist entropy in the sheared TCs

The budget analysis in section 3 indicates that shear-induced asymmetries are crucial for the inner-core moist
the budget results do not explicitly show the physical processes behind the eddy advections. In this section, the detailed contributions associated with the shear-induced eddy to the moist entropy change in the TC will be described. The potential connection between inner-core moist entropy change and TC intensity will be briefly discussed later in this section. The thermodynamic and dynamical structures illustrated in this section are time-averaged results from 5 to 6 h after the imposition of VWS and are consistent with the long-term-averaged fields (not shown), and thus are considered to be representative of the first 12-h structure.

### a. Entropy reduction in the eyewall and the boundary layer

As suggested by the budget analysis (section 3b), there are two processes that contribute to the reduction in moist entropy in the midlevel eyewall: one is the radial eddy flux of low entropy as suggested by TE10, and the other is the vertical eddy flux. To gain an insight into the reduction of moist entropy in the midlevel eyewall, Fig. 7 shows the vertical cross section of the moist entropy anomaly as well as the radial and vertical motion anomalies along the azimuth in SH10. These anomalies are averaged over a radius of 40–60 km. At midlevels, relative inflows coincide with the lower-entropy region, while outflows are mostly found in the place of higher-entropy region (Fig. 7a). Thus, the relative flows could bring the lower-entropy air into, and take the higher-entropy air out of, the eyewall at the midlevels. On the other hand, the shear-induced asymmetries organize the lower (higher)- air in the downshear-right (left) quadrant from the mid- to upper levels, where the relative downward (upward) motions are dominant (Fig. 7b). Consequently, the relative downward motions take the low entropy to the midlevels, while the updrafts carry high entropy to the upper levels. Therefore, the midlevel ventilation is not only the result of radial eddy advection but also the consequence of vertical eddy advection.

To further illustrate how the midlevel ventilation is accomplished by both the radial and vertical eddy fluxes, the radial and vertical distribution of moist entropy eddy fluxes together with the asymmetries of radial and
vertical winds averaged from hour 5 to hour 6 after VWS was imposed are elucidated in Figs. 8 and 9, respectively. Figure 8a shows the radial distribution of radial eddy flux averaged between 4 and 6 km in height, where the midlevel ventilation usually occurs. Figure 9a demonstrates the vertical distribution of vertical eddy flux averaged over a radius of 40–60 km, where the eyewall is usually located at midlevels. In SH00, the radial eddy flux is concentrated between a radius of 60 and 80 km (Fig. 8a), and the vertical eddy flux focuses at low levels below a height of 4 km (Fig. 9a). In shear cases, the radial eddy flux is much enhanced (except for SH05, when the magnitude remains unchanged) and covers a radius of 40–120 km (Fig. 8a). Although the vertical eddy flux in the shear flow also has a maximum at low levels (as in SH00), a second maximum is seen at midlevels, at about 5.5 km in height, which is not present in SH00, and so can be attributed to VWS (Fig. 9a). Therefore, the moist entropy reduction could be induced by vertical eddies and usually occurs at the midlevels, given that \(-\partial(w' S')/\partial z\) approximately represents the local moist entropy tendency resulting from the vertical eddy flux. A similar analysis of radial eddy flux also indicates that moist entropy between a radius of 40 and 60 km is reduced by the radial eddy flux (Fig. 8a). The corresponding processes can be seen intuitively in Figs. 8b and 9b.
moist entropy in the vertical direction. As the vertical eddy flux is maximized at a height of 5.5 km (Fig. 9a), the moist entropy in the midlevel eyewall could also be ventilated by the vertical eddy advections. On the other hand, above the height of 5.5 km where the vertical eddy flux is maximized, vertical eddy advections result in the local increase of moist entropy (Fig. 3j). Although the magnitude of the vertical eddy flux is a factor of 10 less than that of the radial eddy flux, the vertical scale is also a factor of 10 smaller than the radial scale. Therefore, the net local tendencies of moist entropy resulting from the vertical eddy flux and radial eddy flux are of the same order and both important in the midlevel ventilation. That is to say, both the radial and vertical eddy fluxes, rather than only one, should be incorporated together when considering the three dimensionality of the sheared TC. Note that the vertical eddy fluxes could also be parameterized in an axisymmetric model, as in TE10 and Tang and Emanuel (2012); however, the relationship between the vertical eddy fluxes and the radial eddy fluxes must be considered as the flows are governed by dynamical constraints such as mass continuity. With a 3D full-physics numerical model, these eddy fluxes could be clearly demonstrated simply through the model constraints. Therefore, a 3D model offers the advantage of evaluating the ventilation hypotheses of a TC in sheared flows.

In addition to the moist entropy reduction within the midlevel eyewall, the shear-induced convective asymmetries can also frustrate the moist entropy in the BL (Fig. 3b). To elucidate how the moist entropy is depressed in the BL by the VWS, Fig. 10 shows the horizontal distribution of the vertically averaged (between 0 and 1.5 km in height) and time-averaged (5–6 h) entropy downflux, downward motion (Figs. 10a,b), the moist entropy anomaly, and radial flows (Figs. 10c,d) in SH00 and SH10. Here, the definition of moist entropy downflux (the same as in RMN10) is \( \mathbf{\omega} \cdot \mathbf{S}' \), in which \( \mathbf{\omega} \) is downward motion and \( \mathbf{S}' \) is the moist entropy anomaly. Positive values represent the downward flux of anomalously lower entropy into the BL. In the absence of VWS, there is little low-entropy downflux (Fig. 10a), weak entropy anomalies, and nearly axisymmetric radial inflows (Fig. 10c). Compared with SH00, there is a much stronger downdraft, as well as the low-entropy downflux in SH10, which occurs from downshear left at a radius of 90 km to upshear at about 30–60 km (Fig. 10b). The coincidence of entropy downflux and the low-entropy anomaly (Figs. 10b,d) indicates that most of the moist entropy depression in the BL might originate from the midlevel lower-entropy air brought down by the downdraft, which is consistent with RMN10. Afterward, the lower-entropy air is swept into the eyewall by the strong inflows (Fig. 10d), which are highly asymmetric and organized from downshear right to downshear left.

Although the moist entropy in the BL is lowered by the eddy advections in the case of VWS, this reduction could be partly restored. Figure 11a–c show the time (5–6 h) and vertically (0–1.5 km) integrated PBL tendency of moist entropy \( \mathbf{\omega} \cdot \mathbf{S}' \) in SH00, SH10, and their difference, respectively. The PBL tendency results from the vertical mixing through turbulence and thus could represent the entropy tendency from the surface.
Figure 11d shows the time (5–6 h) and vertically (0–1.5 km) integrated vertical gradient of entropy downflux ($\partial w/\partial z$) in SH10, which represents the net entropy downflux into the BL. As shown in Fig. 11c, the PBL tendency is stronger in SH10 than its counterpart in SH00 in the region where the low-entropy downflux is dominant (Fig. 11d). This is because the reduced entropy in the downshear-left to upshear-left quadrant in the BL is able to enhance the entropy deficit at the surface and result in a stronger enthalpy flux. Although the ocean surface has this ability to restore the BL’s entropy in the presence of VWS, it is not large enough to offset all of the entropy reduction by eddy advection. It can be seen from Figs. 11c and 11d that the low-entropy downflux (Fig. 11d) is larger than the PBL tendency (Fig. 11c). Hence, the moist entropy in the BL will decrease in the first few hours after the VWS is imposed. Over time, this recovery ability continues to strengthen so that it can become strong enough at some point to balance, or even overcome, the entropy depression in the BL.

b. Entropy supply outside the eyewall above the boundary layer

Eddy advection cannot only result in the reduction of moist entropy within the eyewall as described in the above subsection but can also supply the moist entropy outside the eyewall above the BL as shown in Fig. 3b. The shear-induced convection in the downshear quadrant is the driver for the increase in moist entropy outside the eyewall above the BL and also for the entropy depression in the BL, but they occur in different quadrants.

To demonstrate how the VWS increases the moist entropy outside the eyewall above the BL, Fig. 12 shows a plane view of the vertically averaged (between 2 and 4 km in height) and time-averaged (5–6 h) upflux of entropy anomalies and upward motion. The entropy upflux is defined as $w_+ S'$, where $w_+$ is the upward motion and...
$S'$ is the moist entropy anomaly. In SH00, the entropy upflux is mostly concentrated in the eyewall between a radius of 40 and 60 km (Fig. 12a). In the presence of VWS, upward motions are induced outside the eyewall from downshear right to downshear left (Figs. 12c,d), which is consistent with observations (Corbosiero and Molinari 2003; Zhang et al. 2013; DeHart et al. 2014). The positive entropy upflux indicates that these updrafts outside the eyewall are capable of transporting the higher-entropy air upward from the BL, resulting in an increase in moist entropy above the BL. This higher-entropy upflux comes mainly from upward transport of BL moisture (not shown). Although it is asymmetric, this higher-entropy upflux is strong enough to be projected onto the azimuthal-mean component, which might have some impact on TC evolution at the vortex scale. Its azimuthal-mean effect can be indicated from the budget results (Figs. 3b, 3j, and 4b), which show that the moist entropy outside the eyewall above the BL increases and the radial gradient of moist entropy is reduced.

The updrafts outside the eyewall might be induced by balanced dynamics. Figure 13 shows the entropy anomaly, positive vertical motion anomaly, and asymmetric storm-relative flows in the BL below a height of 1.5 km in SH00 and SH10 (5–6 h). In the absence of VWS (SH00; Fig. 13a), the upward motion anomaly is only concentrated in the eyewall and the asymmetric storm-relative flows are much weaker. In SH10 (Fig. 13b), the upward motion is collocated with a positive entropy anomaly from downshear-right to the downshear-left quadrant, which is consistent with observations (Zhang et al. 2013). Given that the entropy decreases with height in the BL outside the eyewall, this positive entropy anomaly indicates the presence of upward distorted isentropes, which are possibly the balanced response of a tilted vortex as indicated by the balanced dynamics (Jones 1995). Hence, the upward motion in the downshear-right sector outside the eyewall might be initiated as the circulation flows along the distorted isentropes as suggested by the dry dynamics. In the presence of moist convection (in the presence of

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**Fig. 11.** The integrated PBL entropy tendency (J kg$^{-1}$ K$^{-1}$ s$^{-1}$) and vertical gradient of entropy downflux (\(\partial_w S'/\partial z\), J kg$^{-1}$ K$^{-1}$ s$^{-1}$) from the surface to the 1.5-km height and over the 1-h time interval (5–6 h) for (a) SH00, (b) SH10. (c) Difference between (b) and (a), and (d) entropy downflux for SH10. The values have been divided by 1000. The arrows at the bottom right of (b) and (d) denote the shear vector.
diabatic heating), the flows would penetrate through the isentropes instead of being strictly along the isentropes. Another explanation for the initiation of updrafts in the downshear-right quadrant relates to vorticity balance on the vortex scale (Willoughby et al. 1984; Bender 1997). As shown by the asymmetric flows in Fig. 13b, strong radial inflows occur in the downshear-right sector. The storm-relative inflows in this region are contributed by the right-of-shear motion through the interaction between upper- and lower-level circulations in a tilted vortex. As the vorticity decreases with radius outside the eyewall, the asymmetric inflows in this sector would result in a negative radial advection of vorticity. The vorticity conservation argument requires a positive stretching tendency to balance the negative radial advection and thus leads to the upward motions.

Therefore, governed by the balanced dynamics, the organization of convection by the VWS cannot only flush the low-entropy downflux into the BL at downshear left to upshear left but can also transport the high moist entropy upward into the region above the BL outside the eyewall from downshear right to downshear left, resulting in a decrease of the radial gradient of moist entropy across the eyewall.

c. Implication of inner-core entropy change for TC intensity

As shown in Fig. 1b, the evolution of TC intensity is closely connected to the radial gradient of moist entropy across the eyewall. In the absence of VWS, the TC intensifies as the radial gradient of entropy increases, and the storm remains steady when the entropy difference only varies a little. After the imposition of moderate to strong VWS (SH10 and SH15), the TCs weaken significantly and the radial gradient of moist entropy decreases rapidly. Therefore, the impact of VWS on TC intensity can be explored by examining the inner-core entropy change, especially the radial gradient of moist entropy across the eyewall. Based on the framework of TC energetics, the energy for the TC heat engine is proportional to the area enclosed by the secondary circulation thermodynamic path in a T–S diagram (see Fig. 2b).
in TE10 and Fig. 2 in RMN10). The strong VWS can reduce the radial gradient of moist entropy across the eyewall and, thus, modify the thermodynamic path to decrease the enclosed area in the T–S diagram, hence weakening the TC intensity. As shown in the above results, the radial gradient of moist entropy across the eyewall may be reduced by two different mechanisms: one is a decrease in moist entropy within the eyewall as indicated by TE10 and RMN10, and the other is the supply of moist entropy outside the eyewall above the BL. This potential new pathway could also lead to a decrease in the enclosed area by the secondary circulation thermodynamic path in the T–S diagram (as shown in Fig. 2b in TE10), resulting in less mechanical energy available to balance the dissipation and subsequent TC weakening. Such a moist entropy increase outside the eyewall above the BL results from the vertical eddy flux of high entropy, which is mostly contributed by the upward transport of moisture through the shear-induced convection. Therefore, this indicates that the water vapor outside the eyewall and above the BL might have an important role in TC evolution. However, the potential role of this pathway and its robustness need to be evaluated more carefully.

5. Summary

This study investigated the detrimental effect of VWS on TC intensity using a set of idealized TC simulations in a 3D, full-physics numerical model. A detailed budget analysis of moist entropy was conducted, in both the quiescent environment and shear flows, to examine the impacts of VWS on the inner-core thermodynamics and intensity change of TCs.

We found that strong VWS plays a negative role in maintaining the radial gradient of moist entropy across the eyewall, thus leading to the TC weakening, mainly through shear-induced convective asymmetries. Therefore, the impact of VWS on TC intensity could be presented by the inner-core moist entropy change, which is primarily controlled by three pathways.

In the BL inflow layer, the midlevel lower-entropy air is flushed into the BL by strong downshear-left asymmetric downdrafts and then swept into the eyewall through the BL inflow, which is similar to the mechanism proposed by RMN10 and the observations of Zhang et al. (2013). The BL’s eddy advections extend far outside the eyewall and are responsible for the reduction of moist entropy in the BL.

When the secondary circulation transports moist entropy into the eyewall, it is further frustrated by the VWS-induced asymmetries at midlevel. The asymmetric convection is organized in such a way that the lower-entropy air is located in the region where the radial inflow and relative downward motion dominate while the higher-entropy air coincides with the outflow and updrafts. Therefore, not only does the radial eddy flux intrude the lower-entropy air into the eyewall, as emphasized by the previous studies in an axisymmetric model (TE10; Tang and Emanuel 2012), but the vertical eddy flux can also frustrate the moist entropy in the midlevel eyewall. This suggests that a complex 3D numerical model will be required to evaluate the ventilation hypotheses.

In addition to the above two paths for the midlevel ventilation, an increase in moist entropy is found outside the eyewall above the BL after the imposition of VWS, which is mostly contributed by the convection outside the eyewall initiated by the VWS. Vertical eddy advection associated with these convective activities transports higher-entropy air (mostly contributed by moisture) upward into the region above the BL and then...
further decreases the radial gradient of moist entropy across the eyewall. This feature is believed to be partly responsible for the intensity decrease and to be related to a potential new pathway that restricts TC intensity in the vertically sheared environmental flow. The corresponding results will be presented in due course.

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APPENDIX

Moist Entropy Including Ice, Its Simplification, and Governing Equation

Following Emanuel (1994), the total specific moist entropy (entropy per unit mass of dry air) for an air parcel including dry air, water vapor, liquid water, and ice could be defined as

$$ S = S_d + q_v S_v + q_l S_l + q_i S_i, $$

where $S_d$, $S_v$, $S_l$, and $S_i$ are the specific entropy for dry air, water vapor, liquid water, and ice; and $q_v$, $q_l$, and $q_i$ are mixing ratios for water vapor, liquid water, and ice, respectively. The total specific moist entropy could be further written as

$$ S = S_d + q_i S_l + q_v(S_v - S_l) + q_i(S_l - S_i). $$

(A2)

using the relationship $q_i = q_v + q_l + q_i$. The entropy of dry air and liquid water are

$$ S_d = c_p \ln T - R \ln p_d \quad \text{and} \quad S_l = c_l \ln T. $$

(A3)

The entropy changes during the phase transition of vaporization and melting (Pointin 1984) are given by

$$ S_v - S_l = \frac{L_v}{T} + c_l \ln \frac{T}{T_l} - R v \ln \left( \frac{H_l}{H_i} \right) \quad \text{and} \quad (A5) $$

$$ S_l - S_i = \frac{L_f}{T} + c_l \ln \frac{T}{T_l} - R v \ln \left( \frac{H_i}{H_l} \right). $$

(A6)

If we assume that the temperature of water vapor, liquid water, and ice in the air parcel are the same, the total specific moist entropy could be written as

$$ S = (c_p + c_l q_v) \ln T - R \ln p_d + \frac{L_v q_v}{T} - \frac{L_f q_i}{T} - R v(q_v + q_i) \ln \left( \frac{H_i}{H_l} \right) + R v q_i \ln \left( \frac{H_i}{H_l} \right). $$

(A7)

It can be proved that Eq. (A7) is equivalent to Eq. (3.13) in Hauf and Holler (1987) after some manipulations. There might be some difficulties when this formulation is used for budget analysis of moist entropy from the WRF output. First, the latent heat of vaporization $L_v$ and melting $L_f$ are functions of temperature, which makes the calculation of entropy inconvenient. Second, when taking the derivative with respect to time, the second term $c_l q_v [(d \ln T)/dt]$ could not be explicitly calculated since the WRF Model does not provide the physical tendencies for the absolute temperature. Therefore, simplifications to the formulation are made, following the idea of B08, to facilitate the budget calculations.

From the first law of thermodynamics including the ice processes (Tripoli and Cotton 1981) we have

$$ c_{pm} d \ln T - R_m d \ln p + \frac{L_v}{T} dq_v - \frac{L_f}{T} dq_i = 0, \quad (A8) $$

where $c_{pm} = c_p + c_v c_{pv} + c_l c_{pl} + c_i c_{pi}$, and $R_m = R + q_v R_v$; $c_{pv}$ is the specific heat of water vapor at constant pressure; $c_l$ is the specific heat of ice; and $c_{pm}$ is the specific heat of moist air.

The latent heat of vaporization and freezing could be written relative to a constant value $L_{v0}$, $L_{f0}$ as

$$ L_v(T) = L_{v0} + L_v^*(T) \quad \text{and} \quad L_f(T) = L_{f0} + L_f^*(T), $$

and then Eq. (A8) could be rearranged as

$$ c_p \left( 1 + \frac{c_{pv}}{c_p} + \frac{c_l}{c_p} + \frac{c_i}{c_p} \right) d \ln T - R \left( 1 + \frac{R_v}{R} \right) d \ln p = -\frac{L_{v0}}{T} \left( 1 + \frac{L_v^*}{L_{v0}} \right) dq_v + \frac{L_{f0}}{T} \left( 1 + \frac{L_f^*}{L_{f0}} \right) dq_i. \quad (A9) $$
Using the thermodynamic constants in the WRF Model and with the characteristic value of $q_v \sim 0.01 \text{ kg kg}^{-1}$, $q_l \sim 0.01 \text{ kg kg}^{-1}$, and $q_i \sim 0.001 \text{ kg kg}^{-1}$, we have $e_1 \approx 1.06$, $e_2 \approx 1.02$. If we choose some appropriate value of $L_{o0}$, $L_f0$ to make $e_1 \approx e_3 \approx e_4$ as in B08, we could write Eq. (A9) as

$$c_p \frac{d \ln T}{dT} - R_e \frac{d \ln p}{dT} = - \frac{L_{o0} \delta_v}{T} d q_v - \frac{L_f0 \delta_l}{T} d q_l,$$

Since $e_2/e_1 \approx 0.96 \sim 1$, the thermodynamic equation could now be expressed as

$$dS = c_p \frac{d \ln T}{dT} - R d \ln p + \frac{L_{o0} \delta_v}{T} d q_v - \frac{L_f0 \delta_l}{T} d q_l. \tag{A10}$$

We set $L_{o0}$ to be $2.555 \times 10^6 \text{ J kg}^{-1}$ as in B08 and $L_f0$ to be $2.832 \times 10^5 \text{ J kg}^{-1}$, which satisfies the approximation assumptions at very low temperatures. Notice that Eq. (A7) is reversible, while Eq. (A10) is pseudoadiabatic.

Integrate Eq. (A10) from 1 K, 1 Pa and with the aid of the Clausius–Clapeyron equation and the Kirchoff’s relation, an approximate formulation for the moist entropy including ice processes could be obtained:

$$S = c_p \ln T - R \ln p + \frac{L_{o0} \delta_v}{T} - \frac{L_f0 \delta_l}{T}$$

$$- R_e (q_v + q_l) \ln (\mathcal{H}_l) + R_e q_i \ln (\mathcal{H}_i), \tag{A11}$$

which is consistent with Eq. (2) in section 3a. Note that we use the total pressure $p$ instead of $p_d$ in Eq. (2) since the entropy could be written as a function of potential temperature (see below) by doing this and the WRF model output could be used directly.

Adding a constant $R \ln p_0$ to both sides of Eq. (A11) and using the definition of potential temperature, the moist entropy could be written as

$$S = c_p \ln \theta + \frac{L_{o0} \delta_v}{T} - \frac{L_f0 \delta_l}{T} - R_e (q_v + q_l) \ln (\mathcal{H}_l)$$

$$+ R_e q_i \ln (\mathcal{H}_i) - R \ln p_0. \tag{A12}$$

Taking the total derivative of both sides of Eq. (A12) with respect to time and with the help of the Clausius–Clapeyron equation, the Kirchoff’s relation, and the definition of relative humidity, we have

$$\frac{dS}{dt} = c_p \frac{d \theta}{dt} + \left( \frac{L_{o0}}{T} - R_e \ln (\mathcal{H}_l) + 1 \right) \frac{d q_v}{dt}$$

$$- \left[ \frac{L_f0}{T} + R_e \ln \left( \frac{\mathcal{H}_l}{\mathcal{H}_i} \right) \right] \frac{d q_i}{dt} - R_e (q_v + q_l)$$

$$\times \frac{d \ln e_s}{dt} - \frac{d \ln q_d}{dt} + R_e q_i \left( \frac{d \ln e_s}{dt} - \frac{d \ln q_d}{dt} \right), \tag{A13}$$

where $e_s$ and $e_d$ are saturated water vapor pressure over liquid water and ice surface, respectively, and $q_s$ and $q_i$ are saturated mixing ratio of water vapor over liquid water and ice surface, respectively. Using the definition of mixing ratios $[q = (R/R_e)(e/p_d)]$, we can finally get the governing equation of moist entropy:

$$\frac{dS}{dt} = c_p \frac{d \theta}{dT} + \left( \frac{L_{o0}}{T} - R_e \left[ \ln (\mathcal{H}_l) + 1 \right] \right) \frac{d q_v}{dT}$$

$$- \left[ \frac{L_f0}{T} + R_e \ln \left( \frac{\mathcal{H}_l}{\mathcal{H}_i} \right) \right] \frac{d q_i}{dT} - R_e q_i \frac{d \ln p_d}{dT}. \tag{A14}$$

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